Mathematical Modeling in the Geosciences
GEOSC 561

Instructors: R. Slingerland & L. Kump

Required Text:

A. *Mathematical Modeling of Dynamical Systems: A Primer* (2011*)* Slingerland, R., and L. Kump, Princeton University Press Published by Princeton University Press, 41B William Street, Princeton, New Jersey 08540

Supplementary Texts:

A. *Computational Techniques for Fluid Dynamics,* Vol. 1, Fletcher, C. A. J., Springer-Verlag Series in Computational Physics, 2nd Edition, 1991

B. *Numerical Methods in the Hydrological Sciences,* George Hornberger and Patricia Wiberg, Special Publication Series 57, American Geophysical Union, 2005 [This is an e-book available from AGU by pdf download].

C. Some Matlab manual such as: *Mastering Matlab 7: A Comprehensive Tutorial and Reference,* Hanselman and Littlefield, Prentice Hall, 2005.

D. *Simulating Clastic Sedimentary Basins***,** Slingerland, Harbaugh, and Furlong, Prentice Hall. 1994.[1]

E. *Partial Differential Equations for Scientists and Engineers,* S. J. Farlow, 2nd Edition, New York, Wiley. 1982.

F. *Ordinary Differential Equations, an elementary textbook for students of mathematics, engineering, and the sciences,* M. Tenenbaum and H. Pollard. New York, Harper & Row. 1963.

E. *Numerical Partial Differential Equations,* Thomas, J. W., Vol. 1, Springer Texts in Applied Mathematics 22, 1995.

F. *Computational Techniques for Fluid Dynamics: A Solutions Manual,* Fletcher, C. A. J. and Srinivas, K., Springer-Verlag, 1992.

G. *The Nature of Mathematical Modelling*, Gerschenfeld, Niels, Cambridge University Press, 1999.

H. *Numerical Methods for Engineers,* Chapra, S. C., Canale, R. P., McGraw Hill, 2002.

I. *Computational Fluid Dynamics for Engineers*, Hoffmann, K. A. and Chiang, S. T., Vol. I, 4th ed., Engineering Education Systems, 2000.

J. *Student Guide to CFD* , Hoffmann, K. A. Dietiker, J. F., and Chiang, S. T., Volume III, Engineering Education Systems, 2001.

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[1] Out of Print and Amazon doesn't have used copies available at present. But, Slingerland now owns the copyright .

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Course Outline

Date — *Topic*

Week of Jan 11th: **Basic Modeling and Mathematical Concepts [Ch 1-2]**

What is a model? The power of mathematical modeling; Types of models; The scientist's job: translating reality into modeling; The laws; How differential equations arise; The meaning of the terms set and function; A review of calculus; What do we mean by solution of an equation set?; Value of non-dimensionalizing; Classification of PDEs.

LAB: *Intro to Matlab*

Week of Jan. 18th: **Box Modeling-Unsteady, Uniform Conservation of Mass [Ch 3]**

Concept of mass balance in a homogeneous reservoir; Input, output, steady state; Residence and response time; Flux relationships; Initial value problems; Solving ODEs: Taylor's series expansions, Euler's method

LAB: *TBA*

Week of Jan. 25th: **Box Modeling (cont.)**

Systems of nonlinear ODEs; Coupled reservoirs; Stiff systems, Implicit Methods

LAB: *TBA*

Week of Feb. 1st: **1-D Diffusion Problems [Ch 4]**

Steps in model building; 1st-order rate laws; parabolic PDEs in geosciences; Fick's "Law"; Analytic solutions; Diffusion of solutes in pore waters; Discretization; Obtaining difference operators by Taylor's Series; Criteria of goodness; Scaling relationships

LAB: *TBA*

Week of Feb. 8th: **Some Theoretical Considerations in Modeling PDEs [Ch 2]**

Well-posed problems; Convergence, Consistency, Stability
Initial-boundary value problems; Types of BCs

LAB*: TBA*

Week of Feb 15th: **Multidimensional Diffusion Problems [Ch 5]**

Elliptical problems in geosciences; LaPlace's and Poisson's Equations; Derivation of equations in 2D describing flow in a pumped aquifer

LAB: *TBA*

No Class Feb 22

Week of Feb. 22nd: **Convection-Dominated Problems [Ch 6]**

Derivation and discussion of geological examples; Derivation of equations describing density-driven flows

LAB: *TBA*

Week of March 1st: Convection-Dominated Problems (cont.) [Ch 6]

Upwind differencing, numerical diffusion

LAB: ***Exam I***

Week of March 8th: Spring Break

Week of March 15th: Convection + Diffusion: Transport Problems [Ch 7]

Derivation and discussion of geological examples; Solution schemes;

LAB*: TBA*

Week of March 22nd: Transport Problems with a Twist: Momentum [Ch 8]

Derivation of equations for fluid flow in 1-D; Burger's Eqn

LAB: *TBA*

Week of March 29th: Systems of 1D Nonlinear Transport Problems [Ch 8]

Gradually varied flow in open channels; Dam Break problem

LAB: *TBA*

Week of April 5th: Nonlinear Hyperbolic Systems: Circulation [Ch 9]

2-D Vertically Integrated Incompressible Flows: Derivation and Solution

LAB*: TBA*

Week of April 12th: Hyperbolic Systems: Solutions [Ch 9 cont.]

LAB*: TBA*

Week of April 19th: Individual Projects

LAB*: Exam II*

Week of April 26th: Individual Projects (cont.)

LAB*: Student Presentations*